LOCAL SCALING PROPERTIES AND MARKET TURNING POINTS AT PRAGUE STOCK EXCHANGE

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We apply a method of time-dependent Hurst exponent, proposed in the series of papers by Grech and Mazur [*Physica A* **336**, 335 (2004)], Grech and Pamula [*Physica A* **387**, 4299 (2008)] and Czarnecki, Grech and Pamula [*Physica A* **387**, 6801 (2008)], on the stock market of the Czech Republic for a period between 1997 and 2009. Our results support the findings of the authors so that the time-dependent Hurst exponent can give some crucial information before a critical event happens on a market. We also discuss some potentially weak points of the method.

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1. Introduction

Detection of crashes of stock markets has significant importance for traders. As the stock markets are complex systems driven by heterogenous agents with different strategies and investments horizons, the task is quite difficult. Such heterogeneity of investors is in close connection with a potential fractal behavior of the markets [1]. As such, the market is characterized by a fractal measure of Hurst exponent $0 < H < 1$, which is also a measure of long-range dependence in an underlying process [2]. Hurst exponent equal to 0.5 indicates either an independent [3] or a short-range dependent process [4]. For $H > 0.5$, the process has significantly positive correlations at all lags and is said to be persistent [5]. On the other hand, if $H < 0.5$, the process has significantly negative correlations at all lags and is called anti-persistent [6].

In the series of papers, [7–9] applied the time-dependent Hurst exponent to Dow Jones Industrial Average (DJIA) index and Polish WIG index and uncovered connection between an evolution of Hurst exponent and coming market crash. In this paper, we apply the same procedure to the stock market of the Czech Republic for the period between 1997 and 2009.

The paper is structured as follows. In Section 2, we briefly describe detrended fluctuation analysis, which we use for Hurst exponent estimation, and basic logics behind the connection between the time-dependent Hurst exponent and market turning points. In Section 3, the results are presented for the critical periods of years 2000, 2005, 2006 and the financial crisis of 2008–2009. In Section 4, we discuss potential drawbacks of the methodology and stress the condition of well defined trends and stability on the market for the method to work properly. Section 5 concludes.

2. Local Hurst exponent and extreme events

There are many estimators of Hurst exponent in the literature — rescaled range analysis [2], detrended fluctuation analysis [10], detrending moving average [11], generalized Hurst exponent approach [12] and others (for more detailed reviews, see [3,13,14]). In our research, we use detrended fluctuation analysis, which was shown to be quite robust to different characteristics of the time series [13], to get comparable results with [7–9]. Let us first briefly describe the method.

Detrended fluctuation analysis (DFA) was proposed by [10] while examining the series of DNA nucleotides. In the procedure, the time series of length T is divided into sub-periods of length υ and a profile (cumulative deviations from a mean) is constructed. A linear fit $X_\upsilon(t)$ of the profile is estimated for each sub-period. A detrended signal Y_υ is then constructed as $Y_\upsilon = X(t) - X_\upsilon(t)$. Fluctuation $F^2_{\mathrm{DFA}}(\upsilon)$, which is defined as an average mean squared error from the linear fit over all sub-periods of length υ, scales as $F^2_{\mathrm{DFA}}(\upsilon) \approx c\upsilon^{2H}$, where c is a constant independent of υ [15].

As DFA is based on the linear fitting and averaging over sub-periods, a minimum sub-period length $\upsilon_{\min}$ as well as a maximum length $\upsilon_{\max}$ needs to be set to avoid an inefficient fitting and averaging. In the research, we use $\upsilon_{\min} = 5$, $\upsilon_{\max} = T/5$ and $T = 215$ to get comparable results with the referenced papers.

To obtain the time-dependent (or local) Hurst exponent, we need to fix the time series length T and move the estimation window of Hurst exponent. By doing so, we get a new time series of "local" Hurst exponents. As Hurst exponent is not only a measure of persistence but can be also interpreted as a measure of mood on the market, it enables us to interpret the evolution of the time-dependent H series. As $H < 0.5$ characterizes the anti-persistent behavior, the decreasing trend of H can be seen as an increasing nervousness on the market. Similarly, the increasing H can be seen as a support of the trend that has just started. Based on these simple logics, [9] and [7] defined the sufficient conditions for a burst of a bubble or simply a strong reversion of a market trend. With a use of moving averages of Hurst exponents with lag of 5 (a trading week) and 21 (a trading month) trading sessions, which we label as H_5 and H_{21}, respectively, the evolution of the market mood can be interpreted. The conditions, which have to be met simultaneously, are as follows:

- the time-dependent Hurst exponent is in a decreasing trend,
- $H_5 \lesssim 0.5\,,$
- $H_{21} \lesssim 0.45\,,$
- local minimum of the time-dependent Hurst exponent reached a value below 0.4 during the similar period as the previous conditions.

When all these conditions are met, the turning point of the market should be near. Authors showed these conditions were met for the most severe crashes of DJIA (1929, 1987 and 1998). Quite interestingly, authors concluded that even if the attacks of 9/11 had not happened, the market would have turned into a decreasing trend [8]. However, the method seems to work only if the market is in clear stable trend where the sentiments of the traders can be represented by the time-dependent Hurst exponent. This condition is more stressed in our application on PX. Obviously, the use of the method is limited to the detection of turning points or crashes which happen due to inner forces inside the trading process and the sentiments of the traders. External shocks to the market cannot be predicted by such approach.

3. Results

We test ability of the time-dependent Hurst exponent to predict significant turning points at Prague Stock Exchange in the Czech Republic, which is represented by PX stock index. The period covered in our research ranges from 7/1/1997 to 12/31/2009 and thus contains several significant peaks and bottoms as well as the current financial crisis. The turning points, which are researched here, are summed in Table I. We can see that there were not any crashes comparable to the ones of 1929 and 1987 in the USA but rather turning points. Behavior during the first several days after the peak was hit were in order of percentage losses compared to the decades losses during the first several days of the mentioned crises.

TABLE I

Crashes at Prague Stock Exchange.

Top	Bottom	First 3 sessions	First 5 sessions	Total drop
27.3.2000	10.7.2000	−5.50%	−7.23%	−26.49%
10.3.2005	16.5.2005	−3.92%	−11.02%	−15.37%
27.2.2006	13.6.2006	−2.73%	−1.62%	−26.37%
29.10.2007	18.2.2009	−0.55%	−2.48%	−67.54%

The evolution of PX values and the time-dependent Hurst exponent is shown in Fig. 1. We can see that not only the index but also Hurst exponent goes through a rapid development with several short and long lasting trends. The exponent values range between 0.35 and 0.7 and the evolution indicates that there is the connection between the time-dependent Hurst exponent patterns and the turning points on the market.

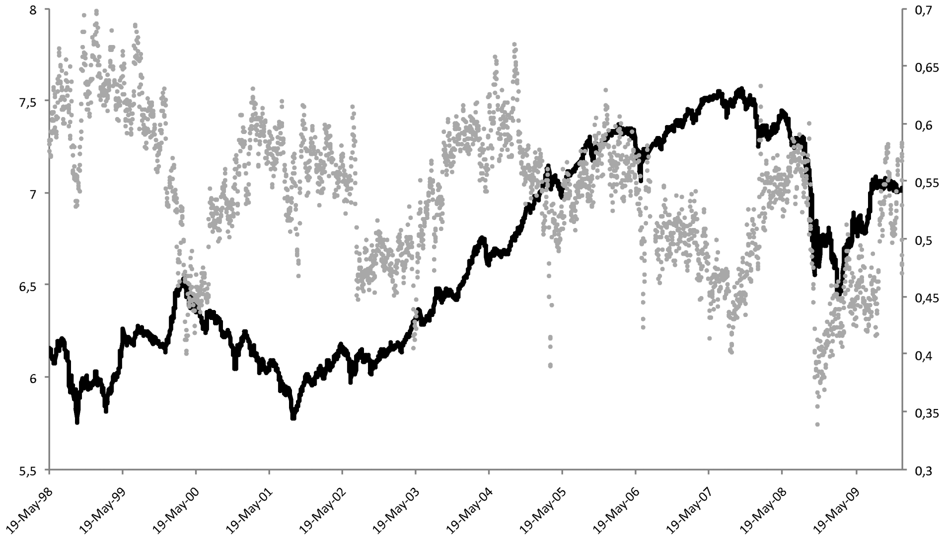

Fig. 1. Evolution of logarithms of PX index (black, on the left y-axis) and time-dependent Hurst exponent (grey, on the right y-axis).

The rest of the section is divided into three parts. In the first part, we deal with the turning points of the years of 2000, 2005 and 2006. The second part is devoted to the current financial crisis. The last subsection discusses the results for different time series lengths T.

3.1. 2000, 2005 and 2006

The first turning point occurred in March 2000 after quite rapid growth which started in November 1999 and was connected with a cumulative return of 38.66% in four months period. The situation before the turning point is illustrated in Fig. 2. The time-dependent Hurst exponent is in a clear decreasing trend and its values reach a minimum of less than 0.4. The conditions for the moving averages are met as well. Therefore, the "crash" signal described in the previous section occurred. In the following three and a half months, the index lost over 26% of its value. However, the decreasing

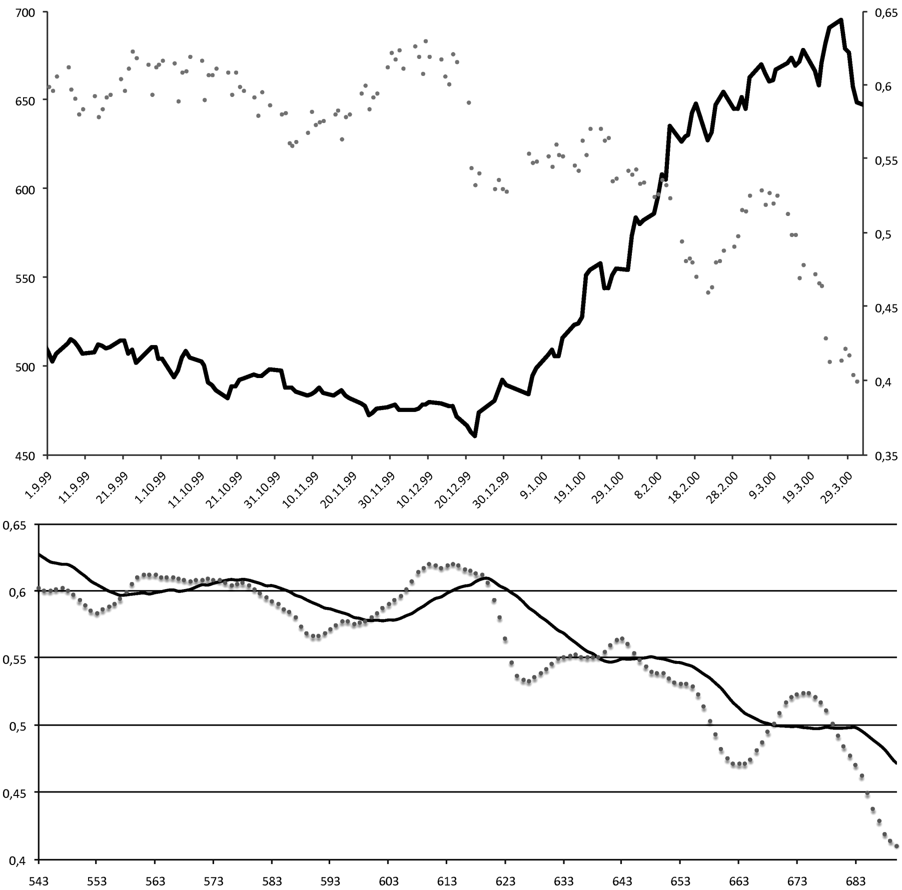

Fig. 2. Evolution of PX index (black solid) and time-dependent Hurst exponent (grey dashed) in the upper chart. Moving averages of time-dependent Hurst exponent with a window of 5 trading days (grey dashed) and 21 trading days (black solid) in the lower chart. Charts show the situation before the turning point of 2000.

trend lasted even longer and was reversed as late as 17th September 2001 when the market hit the bottom of 320.1 points with a cumulative loss of 77.60% since the peak of March 2000.

The second critical point took place on 3/10/2005 and again followed after a very strong increasing trend which started in July 2004 and was connected with a cumulative return of around 46%. The evolution of the index, the time-dependent Hurst exponent and corresponding moving averages are shown in Fig. 3. The pattern is similar to the previous case — decreasing trend of the time-dependent Hurst exponent with the moving averages around the critical levels. However, the situation that happened afterwards is quite different as we can see a crash rather than a turning point of the market as there were significant losses in several sessions right after the peak but not a long lasting trend.

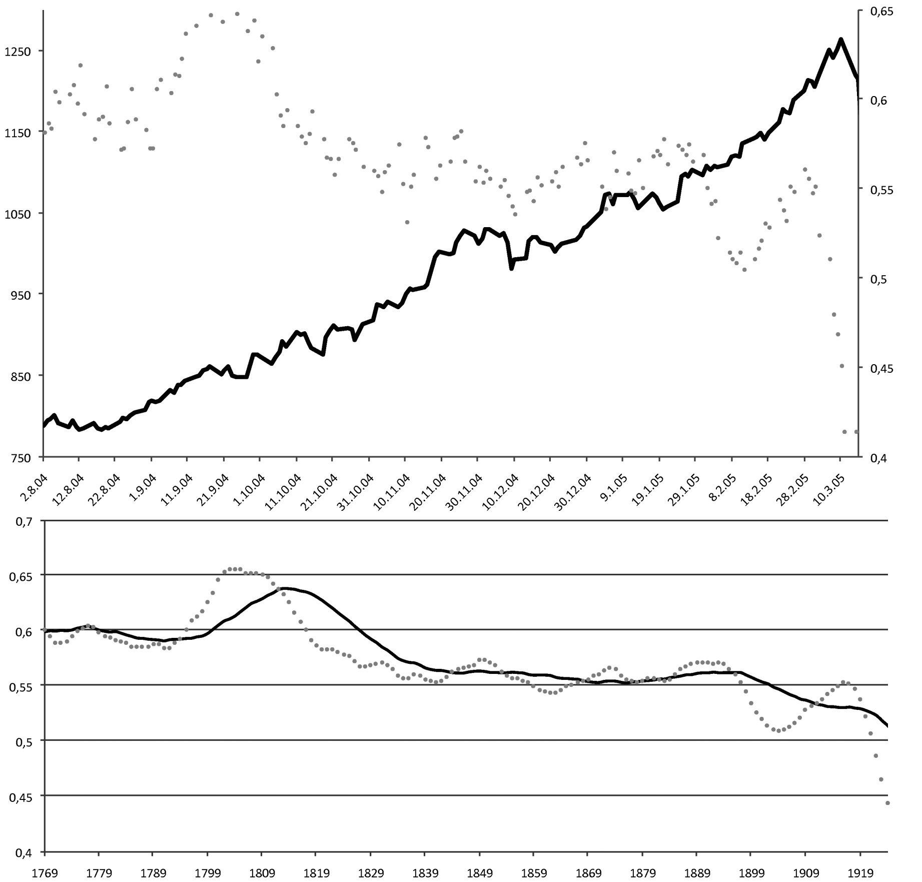

Fig. 3. Charts show the situation before the turning point of 2005. The notation holds.

The third turning point is the one of 2006. The detailed description of the situation is shown in Fig. 4. Even though the turning point was quite strong with a cumulative loss of more than 26% in three and a half months, there is no pattern visible in neither the moving averages of the time-dependent Hurst exponent nor the Hurst exponent itself. However, the turning point followed after several strong corrections between September and November 2005 as well as a small correction in June 2005. Such result confirms the findings of [7–9] who also asserted that the time-dependent Hurst exponent is able to detect critical points only in a presence of stable market trends. This was the case of the first two turnings which we analyzed but is not the case for the critical point of 2006. Moreover, the period of the end of 2005

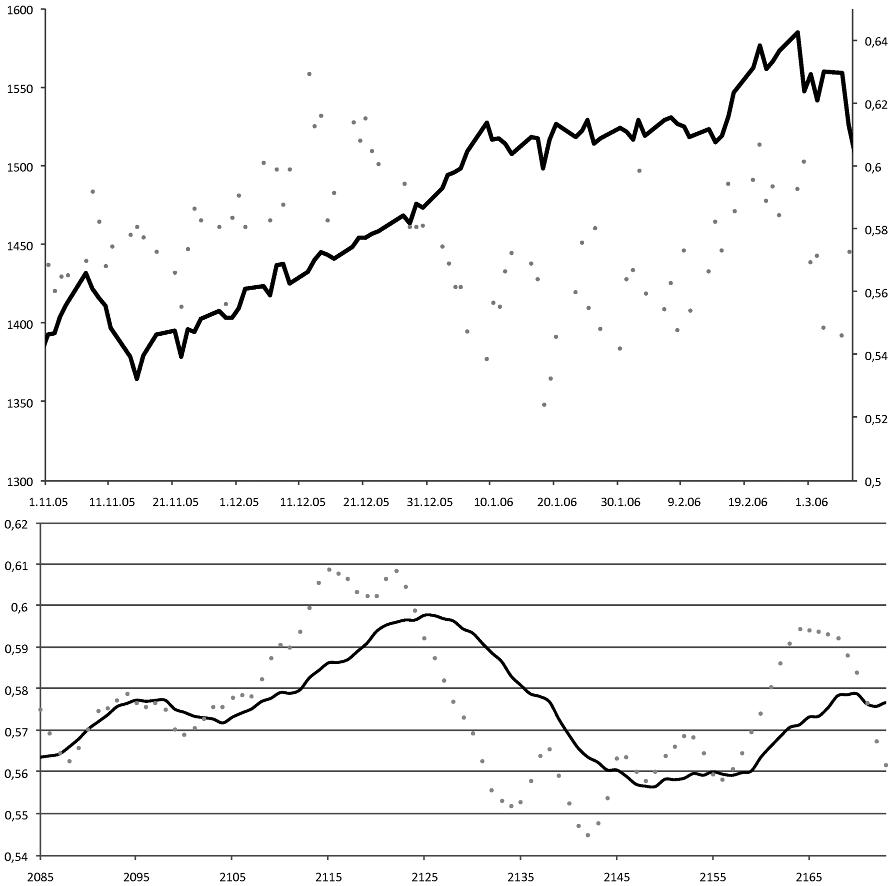

Fig. 4. Charts show the situation before the turning point of 2006. The notation holds.

and the beginning of 2006 was a starting point of long lasting and strong decreasing trend of the time-dependent Hurst exponent which evolved into the biggest market plummeting of the history of PX index.

3.2. Financial crisis of 2008–2009

The current financial crisis is absolutely unprecedential in the history of the Czech stock market as is the case for almost all stock markets with short history. From the peak at the end of November 2007 to the bottom in February 2009, PX index lost over 67% of its value. One of the main challenges of the paper is whether the time-dependent Hurst exponent could have predicted the downturn.

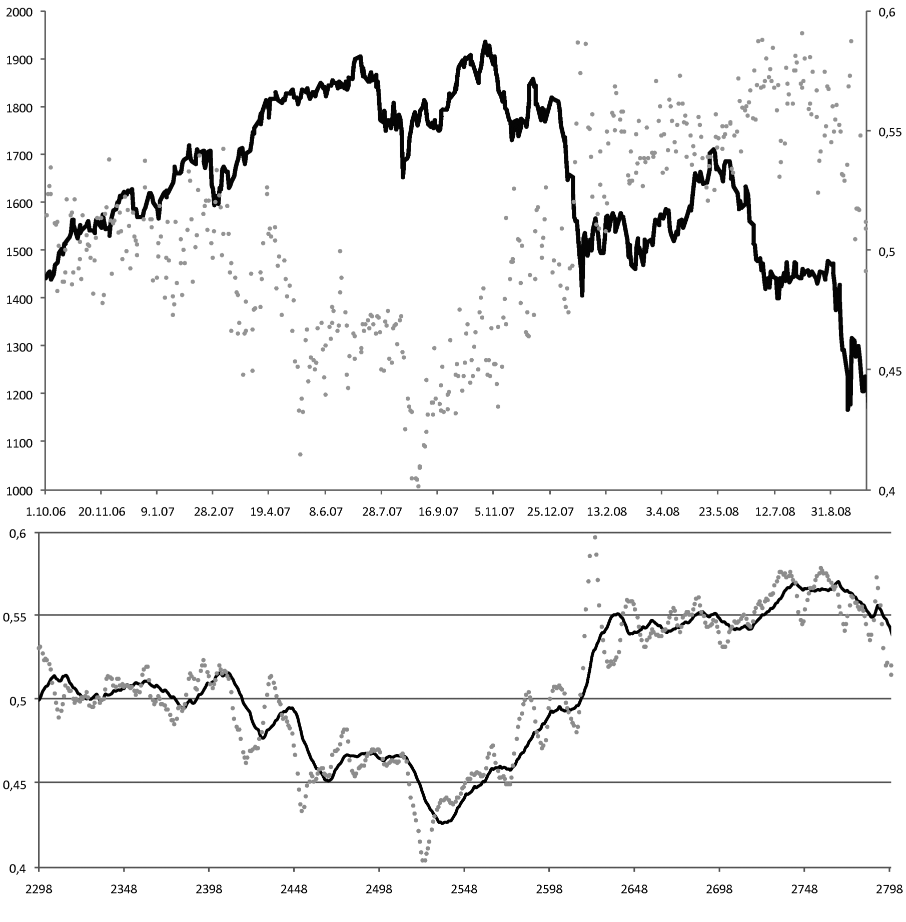

Fig. 5. Charts show the situation before the turning point of the current financial crisis of 2007–2009. The notation holds.

The evolution of the index together with the local Hurst exponent and the moving averages are summarized in Fig. 5. We can see that the "before crisis" pattern appeared with all the criteria strongly met (H_5 very close to 0.4 and H_{21} well below 0.45). Again, the signal showed that the turning point is about to happen in several upcoming trading days. When we turn back to Figure 1, we can see that the decreasing trend of the local Hurst exponent started as early as in November 2005 and thus lasted over two years. This shows that even though the market was strongly growing (with gains of over 30% with the downturn of more than 26% in 2006 being included), the mood on the market was decreasing and the investors were becoming more nervous. After the end of the strong increases, the market turned into a strongly decreasing trend while the evolution of the local Hurst exponent only confirmed that the trend is about to last. As already mentioned, the cumulative losses soared up to more than 67%. The critical of point of 2006 can thus be seen as kind of a "foreplay" before the crisis of 2008–2009.

3.3. Changing time series length T

As DFA has been shown to have large standard errors for small samples, based on Monte Carlo simulations (*e.g.* [16, 17]), we need to check whether the results are robust to a choice of the time series length T. In the previous sections, we used $T = 215$; in Fig. 6, we show the evolution of the local Hurst exponents based on $T = 180, 215, 250, 430$. We also estimated the local exponents for $T = 200$ and $T = 230$, which are not presented in the chart for a sake of clarity. Such lengths were chosen to be close to $T = 215$ so that we can comment on a sensitivity of the method and $T = 430$ was picked to uncover whether there is a significant difference between lengths around one trading year and two trading years.

The results are quite straightforward. Estimates of the local Hurst exponents for the shorter estimation periods T are very similar for the whole sample. Even though the series are quite noisy, the most important trends are kept the same. When looking for the turning point patterns, the results are very similar for all of $T = 180, 200, 215, 230, 250$. For $T = 430$, most of the variation is lost and there are no significant and quickly changing trends so that the method of the local Hurst exponent can be hardly used for the detection of critical points on the market. Such result supports the use of the short time series for such analysis, which is in hand with results of [7–9].

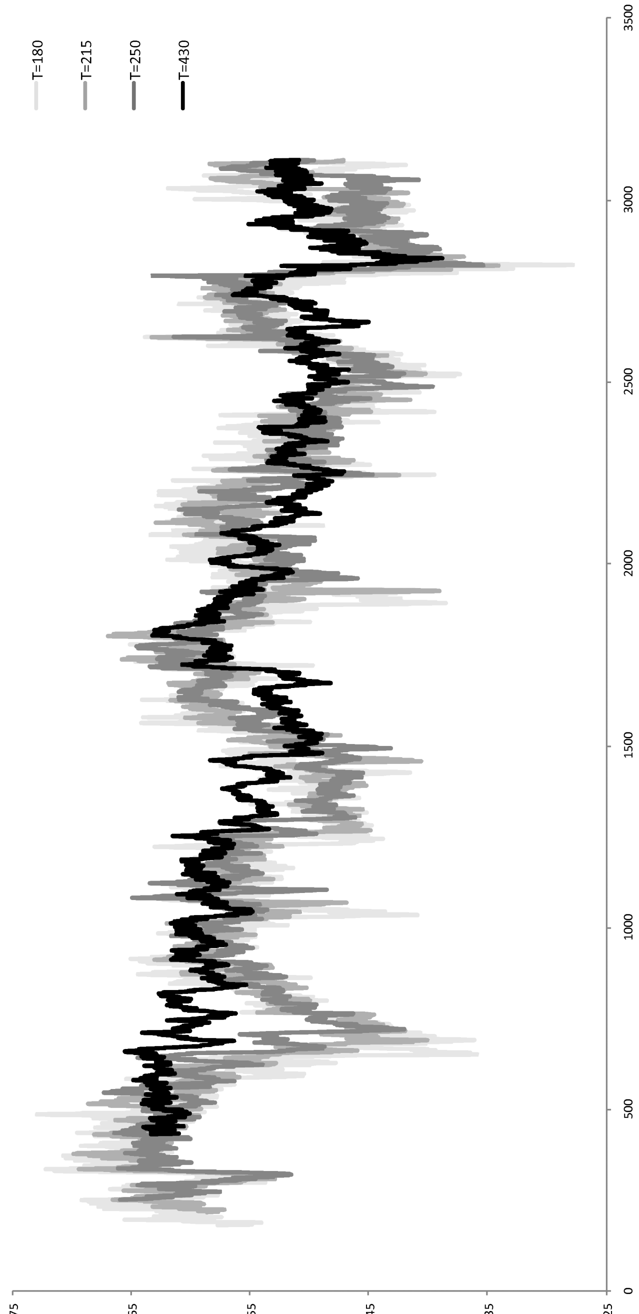

Fig. 6. Comparison of evolution of local Hurst exponents for different lengths T.

To further illustrate the difference between the estimates of local Hurst exponents with different T, we examine correlations between the estimates. In Table II, we show the correlations between the local Hurst exponents. The bigger the difference between estimation periods T the lower the correlation. If $T = 215$ is taken as a reference, the correlations are higher than 0.8 for the time series of length around one trading year. However, the correlation between samples of 215 and 430 drops to 0.51. More importantly, we also present the correlations between changes of local Hurst exponents, which gives more information about co-movements of the exponents series. The discrepancy between the short and the long series is more profound. For all pairs of $T \leq 250$, the correlations are higher than 0.6 whereas for $T = 430$, all the cross-correlations are lower than 0.3.

TABLE II

Correlations of local Hurst exponents with different T.

	$T = 180$	$T = 200$	$T = 215$	$T = 230$	$T = 250$	$T = 430$
$T = 180$	1.0000	0.8596	0.8249	0.7762	0.7093	0.3702
$T = 200$	—	1.0000	0.8910	0.8577	0.8143	0.4490
$T = 215$	—	—	1.0000	0.9039	0.8595	0.5149
$T = 230$	—	—	—	1.0000	0.8907	0.5640
$T = 250$	—	—	—	—	1.0000	0.6263
$T = 430$	—	—	—	—	—	1.0000

TABLE III

Correlations of changes in local Hurst exponents with different T.

	$T = 180$	$T = 200$	$T = 215$	$T = 230$	$T = 250$	$T = 430$
$T = 180$	1.0000	0.7148	0.6695	0.6503	0.6462	0.1944
$T = 200$	—	1.0000	0.6710	0.7434	0.6150	0.2132
$T = 215$	—	—	1.0000	0.6539	0.6149	0.2370
$T = 230$	—	—	—	1.0000	0.6594	0.2416
$T = 250$	—	—	—	—	1.0000	0.2800
$T = 430$	—	—	—	—	—	1.0000

Moreover, we present basic descriptive statistics of the local Hurst exponents as well as of its first differences in Table IV. Mean values of the exponents as well as of their differences are quite stable with changing time series length T. Importantly, standard deviations of the estimates and the changes are decreasing with varying T. Similarly to [8], we use a measure of statistical uncertainty defined as a ratio between the standard deviation and the average of the local Hurst exponents or the differences. There are no strong outcomes from the uncertainty ratio of the local Hurst exponents but there are for the differences. The ratio is strongly varying with T and reaches its local minimum for $T = 215$. Even though the ratio is much lower for $T = 430$, such time series length has already been discussed to lose majority of its variation. Such result further supports the choice of $T = 215$. To summarize, only short estimation periods should be used in the method as longer periods (two trading years and more) do not show enough variation and strong trends.

TABLE IV

Descriptive statistics of local Hurst exponents with different T.

	$T = 180$	$T = 200$	$T = 215$	$T = 230$	$T = 250$	$T = 430$
Mean	0.5399	0.5386	0.5385	0.5381	0.5377	0.5480
SD	0.0679	0.0641	0.0615	0.0597	0.0579	0.0479
Mean of differences	0.00006	0.00007	0.00008	0.00003	0.00005	0.00019
SD of differences	0.0216	0.0167	0.0139	0.0133	0.0116	0.0144
Uncertainty	0.1257	0.1191	0.1141	0.1109	0.1077	0.0874
Uncertainty in differences	392.22	227.37	177.08	513.58	218.78	75.81

4. Discussion

Even though the technique of the detection of upcoming turning points seems to work, it raises a crucial question — Is it not only a coincidence? We try to answer the question with the following example.

Figure 7 shows the evolution of the time-dependent Hurst exponent which characterizes a behavior of a random series based on the returns of PX in the researched period. The random series was simply generated from the shuffled logarithmic returns of PX index which were cumulated to form new time series. The series is purely random but has the same distribution of returns as the original PX index. Moreover, as the initial value of the index was kept the same as for the original series, the last values equals to the original one as well.

We can see that the values of Hurst exponent vary between 0.35 and 0.65 which is less than for the original time series. Nevertheless, H undergoes strong trends comparable to the original series, which is implied by the nature of DFA (there are strong auto-correlations in the differences of the time-dependent Hurst exponent at the lag equal to $iv_{\min}$ for $i \in \mathbb{N}$ and $iv_{\min} \leq T$). Moreover, the shuffled PX index has several points which could be detected as the turning points. The "pre-crash" patterns can be even

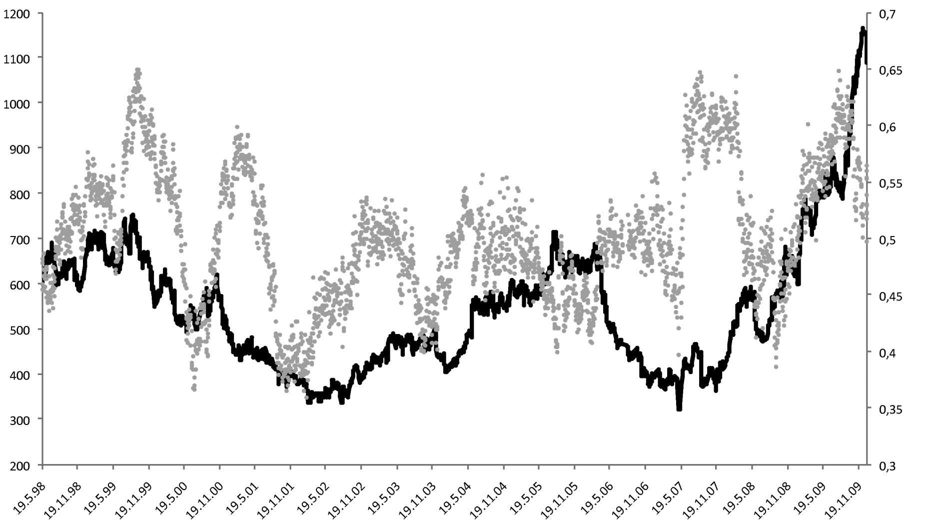

Fig. 7. Evolution of the shuffled PX index (black) and time-dependent Hurst exponent (grey).

found before these artificial critical points. They are artificial because we know that the series is random and therefore, if the time-dependent Hurst exponent pattern signalizes that the critical point is close, it cannot be correct. For the shuffled series, such behavior can be detected in October 2000 or February 2006. However, there is one significant difference in the researched pattern. In the same way as for the turning point of 2006 discussed in the previous section, there are no clear trends and stable behavior present on the market. The shuffled series do not create environment stable enough for the method to be used. As already mentioned by [7–9], the stable and clear evolution of the market is the needed condition for the method of the time-dependent Hurst exponent to be able to detect potential turning points.

5. Conclusions

We applied the method of the time-dependent Hurst exponent, based on the detrended fluctuation analysis, as the crash detection tool proposed by [7–9] on the Prague Stock Exchange PX index. The examined period ranged from July 1997 to the end of 2009. We confirmed that the method works well in the stable market with well defined and long lasting trends. Out of four turning points of PX, three were detected by the method before they happened. Importantly, the method was able to detect the upcoming crisis of 2008–2009. Further, we discussed on some issues of the method such as a choice of estimation period of H. We conclude that the method works well under the set conditions but one must not forget that the method is efficient only for the market with well defined trends and stable behavior.

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